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Book Reviews

BOOK REVIEWS

The field of partial differential equations (PDEs) pervades both pure and applied mathematics; every undergraduate in mathematics, sciences, or engineering ought to get some exposure to it. This usually happens in a course that shows how to solve the classical PDEs by separation of variables. Typically d'Alembert's solution of the wave equation is also presented. These are important analytic techniques. Fewer undergraduates will be exposed to computational techniques, typically in a course on numerical analysis. Rarely will you see both analytic and numerical techniques in the same introductory course.

Our featured review, by Mark Blyth, looks at a new textbook that bucks the trend. *Essential Partial Differential Equations*, by Griffiths, Dold, and Silvester, has a subtitle, *Analytical and Computational Aspects*, which announces that it blends analytic theory and numerical techniques. The authors argue that a study of computational techniques is essential to the understanding of PDEs. I think this is a great idea, and the book looks quite promising from Blyth's review. I haven't yet had a chance to look at it myself, but I hope to do so in the near future.

We have two other reviews of books on PDEs. These are both more specialized and focused on specific classes of numerical methods. One is on finite volume methods for hyperbolic equations. The focus is on hyperbolic conservations laws, which arise in many applications. The other is on reduced basis methods for parametrized PDEs. In the context of Galerkin's method, one seeks to keep the dimension of the trial space relatively small by making "educated" choices of basis functions.

Three of our reviews are on books in mathematical biology. One of them—*Methods and Models in Mathematical Biology*—is a textbook which covers a lot of ground and seems to be suited for advanced undergraduates or graduate students who are well prepared in mathematics. The other two are on hidden Markov processes with applications to biology and an introduction to mathematical epidemiology.

In addition to these we have an introduction to uncertainty quantification and my review of a short introduction to discrete Fourier and wavelet transforms. Once again I wish you happy reading!

David S. Watkins
Section Editor
siam.book.review@gmail.com

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Book Reviews

Edited by David S. Watkins

Featured Review: Essential Partial Differential Equations. *By David F. Griffiths, John W. Dold, and David J. Silvester.* Springer, Cham, Switzerland, 2015. $39.99. xii+368 pp., softcover. ISBN 978-3-319-22568-5.

Partial differential equations (PDEs) is a core subject in any undergraduate mathematics degree and there are numerous books dedicated to the subject already on the market. This new text, which appears under the umbrella of the Springer Undergraduate Mathematics Series (SUMS), is intended as a primer for advanced level undergraduate students meeting the subject for the first time. Its twelve main chapters build from very elementary ideas to more advanced topics: the first nine chapters are suggested as appropriate for an introductory course, and the following three chapters present a fairly technical discussion of finite difference methods that is probably more suited to a higher level course. A thirteenth chapter offers a versatile and interesting range of projects suitable for individual or group work.

The strong recognition of the importance of numerical methods in the modern study of PDEs is a fundamental strength of this new text, which makes it ideal for a first course which aims to complement theoretical study with numerical computation and, importantly, to motivate the student to want to experiment with coding and computation. The authors make clear their opinion, which is shared by this reviewer, that "numerical approximation aspects are central to the understanding of properties of partial differential equations." In the introductory course formed by the first nine chapters, finite difference methods in $\mathbb{R}^1$ make their appearance in Chapter 6. However, the importance of numerical computation as a tool for study and understanding is made clear from the outset. The first figure showing a numerical solution appears on page 5, and thereafter numerical solutions are an integral part of the discourse. The pedagogical value of this is, I think, not to be underestimated. The many computational examples reveal to the student a key aspect of modern scientific practice: computations can throw up quite unexpected behavior that drives deeper thought, deeper investigation, and ultimately, deeper understanding.

The book is written in an engaging and lively style that will appeal to students. In fact, for a book with three different authors, the style is notably uniform. Each chapter begins with an abstract, which is helpful in setting the scene. The brief first chapter gives a quick rundown of some classical PDEs without getting bogged down in too much detail or in derivations (the latter appear in Chapter 3 and are brief). Such brevity is admirable and signals another valuable feature of the text: the authors know when to stop. They have anticipated what students are likely to find difficult, and give details where needed. Examples feature prominently and many of the important ideas are introduced in this way. An extensive list of exercises (300 in total) at the end of each chapter provide a wealth of practice material. Solutions to selected exercises

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are given at the back of the book; the remainder can be obtained by the instructor on request, as can the MATLAB scripts used to generate the numerical results.

A particularly appealing aspect is the skillful course charted between pure mathematical rigor and a more pragmatic, applied mathematical viewpoint. The operator notation introduced in Chapter 2 (and used throughout) gives the book a "purer" feel, but a counterpoint to this is the focus on calculation and interpretation of solutions. The issue of well-posedness appears early and plays a prominent role thereafter. Comparison principles for inverse monotone functions, maximum principles, and energy methods are discussed at some length. Self-adjoint operators and eigenvalue problems are also treated. The classification of PDEs into parabolic, elliptic, and hyperbolic type in Chapter 4, naturally interwoven with an introduction to characteristics, is perhaps the most transparent that I have come across. A more advanced discussion of the method of characteristics for hyperbolic systems of first order PDEs and nonlinear PDEs appears later in Chapter 9, where phenomena such as finite-time singularities, shocks, and expansion fans are introduced.

As noted above, the later chapters provide more technical details on numerical finite difference methods and are perhaps more suited to higher level study. Individual chapters are devoted to finite difference methods for elliptic, parabolic, and hyperbolic equations, respectively, and in that order. Important issues such as the smoothing of discontinuous boundary data or the propagation of discontinuities through into the solution domain are emphasized. Less commonly seen topics include a study of domains with complex geometry. Here it might have been nice to at least signal to students that more advanced techniques (such as boundary element and finite element methods) exist and are surely more suited to such problems, so that the student does not come away with the impression that finite differences are the state of the art here. More mainstream are the issues of consistency, stability, and convergence, which are scrutinized in detail. The material in the chapter on hyperbolic equations is perhaps more challenging and focuses in the main on the appealingly simple-looking, but deceptively difficult, first order equation $u_t + au_x = 0$ for constant a. More advanced topics covered in this chapter include numerical dissipation and flux-limiters.

The stated aim of the Springer SUMS series is to take a "fresh and modern approach" to core foundational material through to final year topics. This book delivers on that promise with great success. Were I about to lecture a course on PDEs, I would reach for it. As a first text that is set at the appropriate level, written in an engaging and stimulating style, which recognizes and incorporates numerical computation as an essential tool for learning and understanding, it looks hard to beat.

MARK BLYTH
University of East Anglia

Solving Hyperbolic Equations with Finite Volume Methods. *By M. Elena Vázquez-Cendón*. Springer, Cham, Switzerland, 2015. $69.99. xviii+188 pp., softcover. ISBN 978-3-319-14783-3.

Whereas almost anyone who has taken an introductory course in numerical solutions to partial differential equations (PDEs) will be familiar with finite difference methods for the standard suite of linear, scalar PDEs (parabolic, elliptic, and hyperbolic), little attention is generally paid to nonlinear systems of PDEs, the most important of which are arguably systems of nonlinear hyperbolic conservation laws. These hyperbolic systems model critical phenomena in fields as diverse as combustion, traffic flow, acoustics, nonlinear waves in elastic media, astrophysics, tsunamis, debris flows, medical shockwave therapy, and so on.

The distinguishing feature of nonlinear hyperbolic problems is that their solutions can develop discontinuities, or *shocks*, in finite time. One of the great advances in numerical methods for hyperbolic conservation laws was made with the realization that using a finite volume discretization along with the solution to a *Riemann problem*, one can reliably capture these physical shocks in numerical schemes [2]. This class of methods, known as *Godunov methods* after the Russian mathematician Sergei Godunov (profiled in this book), is the foundation of many modern numerical methods for hyperbolic conservation laws. The book under review provides the background necessary to understand how these numerical methods work, as well as description and analysis of finite difference, upwind, and finite volume schemes and codes for several of the classic methods in common use.

This English translation of the Spanish text *Introducción al Método de Volúmenes Finitos* (2008) presents a nearly comprehensive, if lean, description of first order numerical methods for one-dimensional hyperbolic conservation laws. I say "lean," because the book, at barely 123 pages, is considerably shorter than other recent texts in the field, including those by E. F. Toro [6], R. J. LeVeque [4], and J. A. Trangenstein [7]. It should not be surprising that this book does not come close to covering all of the material covered by these other texts (all of which are over 500 pages), but its focus on one-dimensional, first order numerical methods makes it a good introductory text to this important class of problems.

Content. This book is nominally ten chapters long, although only Chapters 1, 2, 3, 5, and 6 contain explanatory text. Of the remaining five chapters, Chapters 8 and 9 provide sample MATLAB codes for linear transport and Burgers equations, and Chapters 4, 7, and 10 provide brief biographies of Peter Lax, Sergei Godunov, and Eleuterio Toro, key mathematicians in the development of numerical methods for hyperbolic conservation laws.

Chapter 1 presents the general form of a one-dimensional conservation law and illustrates how a conserved quantity is updated by the difference of fluxes at the ends of a one-dimensional control volume. Several motivating examples are provided, including linear transport, shallow water wave equations (with and without a variable bed), acoustics, Euler equations, and traffic flow. For the constant coefficient transport equation, a solution method using characteristic curves is presented.

In Chapter 2, the reader is formally introduced to the notation used for conservation laws. Most of the examples introduced in Chapter 1 are revisited here and placed in this more formal setting, using the notation for conserved quantities and flux functions that is used throughout the rest of the book. Solutions to a Riemann problem for a constant coefficient, linear system of conservation laws are also given and used to solve the Riemann problem for the linearized shallow water wave equations. Chapter 2 concludes with the only practical mention of conservation laws in more than one space dimension and presents the nonlinear shallow water wave equations in two dimensions.

Chapter 3 presents the most theoretical material of the book, and so is largely independent of the rest. The chapter begins with a motivating introduction demonstrating how a conservation law can generally support discontinuous solutions. The remainder is devoted to four essential theorems on the existence of weak solutions and conditions, known as "entropy conditions," needed to choose the correct weak solution. No proofs are given here, but references to the well-known text by Godlewski and Raviart [1] are provided, as well as a reference to work of Peter Lax [3].

Chapter 5, the longest chapter in the book, introduces the reader to several finite difference and finite volume schemes for scalar conservation laws. After brief discussions on consistency, convergence, von Neumann stability analysis, the CFL condition, and finite difference approximations and truncation error, two centered finite difference schemes are presented: the unstable forward-in-time-centered-in-space scheme and the stable Lax–Friedrichs scheme. Then, the author introduces the upwind scheme and, as she does for the centered schemes, presents detailed convergence and stability analysis. In this section, the connection is made between upwind schemes and finite volume methods. Numerical convergence results are provided for both Lax–Friedrichs and the upwind scheme. Chapter 5 concludes with a discussion of the Godunov method for linear systems.

Chapter 6, the final chapter of actual text, focuses almost exclusively on approximate Riemann solvers, including the well-known Roe solver [5]. Special emphasis is placed on understanding convergence to entropy solutions. Several well-known entropy fixes are proposed for those schemes that do not naturally converge to the correct entropy solution.

Each chapter contains several exercises, many with worked solutions, sprinkled throughout the main text. MATLAB codes presented in Chapters 8 and 9 make concrete the schemes described in the text.

Discussion. In the author's words, this book serves as the "first contact" to finite volume methods, with a "balance between mathematical rigor and the physical intuition that characterizes the very origin of finite volume methods." In this, the book largely succeeds, and while it is not as complete as other texts in the field, it illustrates the basic points needed to implement practical (albeit first order) numerical methods for one-dimensional problems.

The exercises presented in the book all seem very doable by a beginning graduate student and serve as an excellent opportunity for the student to engage with the text.

The MATLAB codes presented at the end of the book (Chapters 8 and 9) provide more opportunities for the interested reader to try out the ideas presented in the text. The codes do not appear to be available online, so to obtain them, one could try contacting the author or just copy them directly from the book. While one would prefer to download the codes, typing them from the book may not be as arduous a task as one might imagine. Only a few codes are longer than a page, and much of the code can be shortened considerably by using MATLAB's extensive vectorization capabilities to eliminate many unnecessary loops.

Finally, the three biographies provided on Peter Lax, Segei Godunov, and Eleuterio Toro provide a welcome glimpse into the lives and accomplishments of three important figures in the development of the numerics and theory of hyperbolic conservation laws.

The main criticism of the book is its lack of any discussion on higher order methods. While the interesting pedagogical development of numerical methods for hyperbolic conservation laws takes place in the context of upwind and Godunov methods, such methods are far too diffusive to be used in practice. On the other hand, a meaningful discussion of higher order methods would then lead to a discussion on the need for limiters, which may have been beyond the scope of the author's intentions for this book. Nonetheless, a brief introduction to the Lax–Wendroff method followed by references to more complete texts would serve the interested student well.

Other material that is absent from the book is a discussion of exact solutions to nonlinear Riemann problems, Riemann invariants, Hugoniot loci, and other topics that would be expected in a more complete text on hyperbolic problems. However, since the book emphasizes the use of approximate Riemann solvers, the absence of these other topics is not critical to the flow of the text, and, in any case, would probably require their own chapters, lengthening the book considerably.

A minor criticism is that the text needs a good copyeditor. One can only applaud the translation team, two former students, Luz Maria García-García and Marcos Cobas-García, from the author's university, for a generally readable text. However, final responsibility for making sure that obvious and nontechnical typos and errors do not appear in the final print copy rests squarely with the publisher, Springer.

I recommend this text to advanced undergraduates eager to get their hands dirty with a computationally focused undergraduate research project, or to graduate students looking for a companion text to go with one of the weightier books on hyperbolic conservation laws. That said, I am tempted to say "wait for the second edition," where I would expect that some of the gaps in explanations and notational inconsistencies, as well as the typos and grammatical mistakes mentioned above, would be fixed. However, then I'd be delaying the rewarding experience that awaits readers when they work to fill in gaps in explanations (never hard to do, in this case) or think carefully about whether the use of different notation from one chapter to the next is intentional (in some cases, yes, it seems) or not. So, while I eagerly await the next printing (and am sending a list of typos to the author), I will surely recommend this edition to my students and anyone else interested in an introduction to hyperbolic equations and finite volume methods.

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Donna Calhoun
Boise State University

Methods and Models in Mathematical Biology. *By Johannes Müller and Christina Kuttler.* Springer, Heidelberg, 2015. $131.00. xiv+709 pp., softcover. ISBN 978-3-642-27250-9.

Between North America and Europe, there are over 40 graduate degree programs in mathematical biology and a growing number of undergraduate degree programs [5]. What topics should be covered in a comprehensive introduction to mathematical biology course? Some examples of broad-scope textbooks I am aware of include *Dynamic Models in Biology* by S. Ellner and J. Guckenheimer (EG) [2]; *Mathematical Models in Biology* by L. Edelstein-Keshet (EK) [1]; and *Mathematical Biology* by J. Murray [3]. In teaching an undergraduate capstone course on mathematical biology, I have used both EG and EK. I find that undergraduates with varied backgrounds respond better to the more mathematically fleshed out description within EK. However, I am always on the lookout for up-to-date texts for senior undergraduate or graduate courses that mix material from stochastic processes, probability, and statistics.

With this in mind, I dug into the recently published *Methods and Models in Mathematical Biology* by Johannes Müller and Christina Kuttler. The core of the book is based on a two-semester graduate course on the mathematical modeling of biological systems. In my opinion, the book is targeted at graduate students with a strong mathematics background. The authors present several of their own personal research interests: ordinary differential equations (ODEs) and bifurcation theory, reaction-diffusion kinetics, stochastic branching processes, and some parameter estimation. Ecology is the most heavily represented area of biology, but some sections near the end cover epidemics, gene networks, and neuroscience. Thus, most of the mathematical topics of the textbook (e.g., compartmental models, Markov chains, discrete dynamical systems, ODEs/PDEs, stochastic processes) are introduced using population dynamics as their motivation. I actually think this approach works well since reproductive/death/migration processes are familiar to students with little biological background, so they can easily visualize their effects and relate them back to the math.

Chapter 1 begins with concepts in compartmental modeling (random variables, simulation of Markov processes and SDEs, parameter estimation, discrete-time dynamics, and continuum limits), with a little discussion of applications. The authors make an effort to interface with lessons in computation, incorporating boxes with pseudocode and some more detailed code in R. One issue I have with this introductory chapter is the unnecessary use of terminology from abstract algebra (e.g., countable index set, semigroup). It would have been better to leave this until later in the text, or to an appendix, to make the introductory chapter a bit gentler. Effects of interactions between different species are introduced in Chapter 2, and classic techniques in bifurcation theory for discrete and continuous dynamical systems are discussed in detail. The authors also touch on nondimensionalization and reparametrization, as well as providing some sample code for simulating dynamical systems in OCTAVE (free GNU alternative to MATLAB) and simbTUM (free GUI developed at TU Munich for simulating and fitting ODE models). While it might have been convenient to have codes in the book all in one language, it is not a bad thing to have students try out and learn a few different software packages. The layout of Chapter 2 felt familiar to me, like the typical phase plane analysis discussions one can find in Chapter 5 of EK or Strogatz's *Nonlinear Dynamics* (Chapter 6). Thus, if such information is familiar to the reader, this chapter can be skipped. With this mathematical background in place, Chapters 3–7 each cover a different topic in mathematical biology: spatial ecology, epidemiology, reaction kinetics, neuronal activity, and evolution. Mathematical topics in these chapters include model derivation, stability analysis, graph theory, pattern formation, and discrete-time dynamics.

Chapter 3 introduces diffusion and advection and leverages spatially-discrete systems to discuss ODE approximations to PDEs. Derivations are accompanied by examples of reaction-diffusion systems from biology-like spreading muskrat populations and the growth of plant pathogen populations. Complementing such classic derivations of continuum models, the authors also discuss moment closure methods for contact processes. I would have liked to have seen more details and exercises on moment closure for stochastic systems, as this is an area that breaks away from more classical approaches in the text. Heterogeneity within populations is addressed in the two concluding sections where age, size, and sex structure are covered from the perspective of compartmental models. Unlike in previous chapters, there is no example code here and less emphasis on computation.

Chapter 4 discusses several variations on the standard susceptible-infected-recovered (SIR) model for infectious disease: optimal vaccination strategies, structured populations, recurrent infection, stochastic models, and random graph models. I would have liked to have seen more discussion of random graph models, such as epidemic evolution on adaptive networks, and more details on the proof of the existence of a giant component, since connectedness seems so pertinent to infectious disease.

Such topics and methods are of growing interest in epidemiology, as well as neuroscience. On the other hand, the authors devote quite a bit of attention to the problem of optimal vaccination strategies. Several pages of functional analysis are devoted to the proof of the existence of an optimal strategy, but most insight comes from the simple examples after the proof. Illustrations of the effects of different vaccination strategies on the infected population would have been helpful too.

Chapter 5 provides a systematic overview of the nonlinear dynamics of gene regulatory networks: time-scale separation for enzymatic reactions, oscillations in negative feedback systems, metastability in mutually inhibitory networks, and the impact of stochasticity. In my opinion, this chapter contains the right balance of schematic illustrations, analyzed example systems, and application to real data. Such material is harder to find in classical math biology texts, although the recent book EG provides some short discussions with a bit of mathematical analysis [2]. I particularly enjoyed reading the section on quorum sensing as occurs in *V. fischeri*: Starting with a schematic of the LuxI-LuxR type regulation system, the authors detail a fully descriptive eight ODE model, which they then reduce to a pair of equations. After analyzing bifurcations and discussing how the feedback in the model engenders hysteresis, they demonstrate parameter fits to real data and incorporate spatial effects. Discussions like these can work well in a graduate course, since they tell a story and cover a variety of topics along the way. The chapter also has a nice variety of mathematical topics, introducing Boolean and Petri network approaches to gene regulation, as well as some classic methods in pattern formation to conclude.

Chapter 6 is a short chapter introducing a few models from neuroscience. Given the limited space, the authors' introduction of the Hodgkin–Huxley, Fitzhugh–Nagumo, and periodic bursting models are reasonable choices since they allow for a discussion of neural excitability along with some fun fast-slow analysis. My main point of contention concerns the exposition on cellular automata (CA) models of large-scale neuronal networks. These approaches have fallen by the wayside in modern modeling of neural systems (I don't think I've ever seen a conference talk involving a CA model of a neuronal network). There are several other large-scale models the authors could have chosen to cover, which would have offered a nice balance of new math and biology concepts: balanced networks, neural field models, coupled oscillator networks, or Fokker–Planck descriptions of integrate-and-fire populations. I would strongly recommend replacing the CA section with one of these topics in the next edition of the text.

Chapter 7 is an even shorter chapter covering evolution. After discussing the Hardy–Weinberg principle, the authors demonstrate the characterization of statistics in a stochastic version of the Hardy–Weinberg model: the Wright model. Adaptation then provides a biological mechanism by which the authors can relate dynamical systems models to evolutionarily stable strategies. The brevity of this chapter leaves one wanting more, and perhaps the authors could have added some fun examples like the side-blotched lizard, whose males interact much like a dynamic rock-paper-scissors game [4]. Evolutionary game dynamics would also have provided an excellent vehicle for introducing dynamics on networks, more discrete and continuous stochastic processes, and adaptive networks.

The book has a very nice diversity of topics in biology and applied dynamical systems. While it is heavy on population dynamics, this can be an advantage when teaching a course, since it keeps novice students focused on modeling concepts before learning about a wider variety of biological systems. A strength of the book is its concerted effort to incorporate parameter estimation into model discussions, which is often swept under the rug in other math biology books. The transition between deterministic and stochastic models is relatively smooth, and this is one of the few broad-scope math biology books I know of that merges deterministic and stochastic analysis along with computation throughout. There is a nice bank of exercises, at the right level for first to second year graduate students. My main suggestion to the authors would be to rebalance some of the mathematical topics, to make it more accessible to graduate students with narrower mathematical backgrounds. Inevitably, mathematical biology courses are populated by students from engineering, biology, physics, and chemistry. As mentioned above, the neuroscience chapter, in particular, could incorporate more modern modeling techniques. More figures would also help, but I know this drastically increases the labor overhead! An interested instructor could build a fine two-semester graduate course around this book, although it would be good to supplement the text with some gentler introductory material from texts like [2, 1, 3]. The authors have the right idea, trying hard to walk the tightrope of (e.g.) merging reduced models, dynamical systems, statistics, computation, and data-fitting, which is what the modern mathematical biologist must do!

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Zachary P. Kilpatrick
University of Colorado

Introduction to Uncertainty Quantification. *By T. J. Sullivan.* Springer, Cham, Switzerland, 2015. $79.99. xii+342 pp., hardcover. ISBN 978-3-319-23394-9.

4book is one of very few that discuss a vast array of topics in the developing field of uncertainty quantification (UQ). In the preface, Sullivan notes that his aim is to "give a survey of the main objectives in the field of UQ and a few of the mathematical methods by which they can be achieved." I find this to be an accurate synopsis of what can be found within.

In the opening chapter, Sullivan nicely introduces the subject of UQ and discusses typical problems that UQ can be used to address. Of particular note, he discusses the immaturity of the field and explains that the current state of research surrounds problems and methods rather than theory. Unfortunately, the remainder of the book places more emphasis on theoretical aspects of topics related to UQ than it does on either problems or methods. The text is mathematically rigorous, and though the intended audience is the senior undergraduate or early graduate mathematics student, a rather thorough understanding of probability and functional analysis is assumed. In the initial chapters, the main ideas from each of these subjects are reviewed; however, lack of prior study would certainly limit the readability of the remainder of the book. As a former student and current professor of computational and applied mathematics, this is a book I might recommend to students as a reference for topics related to UQ, but I would not use it to introduce the subject to them.

Each chapter is more or less self-contained (though not entirely, as there are many references to topics that will be "discussed further in X Chapter"). This makes it possible to skip to subjects of primary interest. Additionally, there is a graphical outline in the beginning of the book that describes the chapter dependencies. Excluding the prerequisite probability and functional analysis chapters, no one chapter depends on more than two (or in one case, three) others. Because of this, however, the fluidity of the content is sometimes lacking, and it can be difficult to anticipate the purpose of earlier discussions. Students will find it helpful to pay close attention to the written outline in the book's introduction to understand how each of the topics fits into the larger story of UQ.

While the first five chapters introduce mathematical subjects that are needed to fully understand the theory of UQ, it is the later chapters of the book that are the most interesting. Sullivan takes great care to explain how each of these later topics is relevant to UQ; the main theorems and how they are used to derive methods; applications; and simple, illustrative examples. He also uses the drawbacks of the more elementary methods to motivate the development of more advanced methods. As an example, the chapter on sensitivity analysis and model reduction is presented extremely well. I found myself wishing that a larger fraction of the text was written similarly.

Overall, this introduction to UQ leaves something to be desired. It is well written, but the emphasis here is placed on introducing the mathematical topics related to UQ (e.g., optimization, orthogonal polynomials, numerical integration) rather than the methods of UQ themselves. It may serve as a survey of some of the mathematical theory behind such methods, but those students and researchers seeking to understand how UQ can be applied to their problems will want to consult another resource. In fact, I am somewhat hopeful that this is the first part of a two part series.

TALEA L. MAYO
University of Central Florida

Hidden Markov Processes: Theory and Applications to Biology. *By M. Vidyasagar.* Princeton University Press, Princeton, NJ, 2014. $59.50. xiv+287 pp., hardcover. ISBN 978-0-691-13315-7.

Vidyasagar's book provides a rigorous introduction to selected topics in the mathematical theory of hidden Markov processes (HMPs) and a sampling of applications of hidden Markov models (HMMs) in biology. The book's specific focus is on two main areas: realization theory. which deals with necessary and sufficient conditions for constructing a stochastic process as an HMP, and large deviation theory for Markov processes, which characterizes the (exponential) rate at which empirical estimates converge to their true values. Additional advanced topics in HMMs covered in the book are ergodicity and alternative representations for HMPs. The coverage of applications in biology is fairly sparse and not necessarily cohesive or representative. The book will appeal primarily to readers interested in the specific advanced concepts in the theory of HMPs that are the focus of the book, whereas readers interested in a broader background in HMMs or particularly focused on biological applications will find the book less useful.

The book is organized as nine chapters grouped into three parts. Part 1, comprising Chapters 1 through 3, provides background in probability and random variables, information theory, and nonnegative matrices. Part 2, comprising Chapters 4 through 7, covers Markov processes, introductory large deviation theory, basic properties of HMPs, and the complete realization problem for HMPs. Part 3, comprising Chapters 8 and 9, covers biological applications. The material in Chapter 1 ("Introduction to Probability and Random Variables") and Chapter 2 ("Introduction to Information Theory") is fairly standard and should be familiar to most readers who have the mathematical background required for the book. Inclusion of this material is beneficial, nonetheless, because it makes the book self-contained and provides required background in a notational convention that is coherent with the rest of the book. Chapter 3 covers the theory of nonnegative matrices, anticipating the close connection between nonnegative matrices and the transition probability matrices for finite state Markov chains. The material in Part 2 is the core material of the book. Chapter 4 develops the theory of Markov processes, making extensive use of the infrastructure of nonnegative matrices developed in Chapter 3. The treatment of ergodicity is one of the advanced concepts included within Chapter 4 that is often missing from basic treatments of Markov processes. Chapter 5 is an introduction to large deviations theory; the key results in this chapter are the rate functions for iid and Markov processes, which derived using the method of types. Chapter 6 covers the basic properties of HMPs. Three alternative representations for defining HMMs are introduced and shown to be mathematically equivalent, and the fundamental problems of inference and parameter estimation for HMMs are discussed. Specifically, given an observed HMP sample, the Viterbi algorithm for obtaining the maximum a posteriori probability (MAP) estimate of the sequence of state transitions is derived and an algorithm is described for (reestimating) the HMM parameters based on the Viterbi estimate. Chapter 8 characterizes the necessary and sufficient conditions for a given stationary stochastic process with a finite sample space of outcomes to be an HMP. The necessary condition is shown to correspond to a finite rank requirement on an infinite Hankel style matrix formed from the statistics of the process. This condition is, however, shown to not be sufficient and a meaningful sufficient condition is obtained only under significant additional assumptions on the process. Chapter 8 initiates a discussion of applications in computational biology. After a very brief introduction to some of the relevant aspects of molecular biology, the basic sequence alignment problem is formulated and a dynamic programming approach is presented for solving the problem. An overview is provided of the GLIMMER and GENSCAN methods, which make use of interpolated Markov models and HMMs, respectively, for gene finding. Finally, the use of profile HMMs for protein classification is briefly reviewed. Chapter 9 covers the theory behind the BLAST algorithm. In particular, the chapter focuses on how the theory of large deviations for i.i.d. processes is utilized in the BLAST methodology to ascertain the significance of high scoring sequence segments found by the algorithm.

HMPs and HMMs have been extensively studied for a long time and there are numerous other publications on the topic of HMMs. It is valuable to compare and contrast this book against those I am most familiar with. The book is almost entirely complementary to the landmark tutorial article by Rabiner [5]. Whereas Rabiner's article focuses entirely on the pragmatic aspects of inference and parameter estimation using HMMs and provides excellent guidance for practical implementation, the present book focuses entirely on the theory without delving into any of the practical aspects. For example, Rabiner's article has an extensive discussion of scaling, which is required in order to address dynamic range limitations of floating point representations in implementations of the forward-backward and Baum–Welch algorithms, whereas the current book does not even outline these algorithms adequately. For bioinformaticians, the standard reference on the topic of HMMs is the book by Durbin et al [3], which is also quite distinct from the current book. Coverage of biological applications is much more cohesive and representative in the Durbin book, whereas the sampling in the present book is much more limited and somewhat haphazard. Specifically, in a strange omission, the basic two-sequence alignment HMM (see [3, Chap. 4]) is not even developed and only the heuristically formulated Needleman Wunsch [4] and Smith–Waterman [7] dynamic programming algorithms are presented for sequence alignment in Chapter 8. The present book does have the advantage over both the Rabiner tutorial and the Durbin book in its coverage of advanced topics in the theory of HMMs. The material on these advanced topics, which is included in Part 2, is largely reproduced from two of the author's published papers [8, 9].

I encountered several errors and typos in the book. The most significant of the observed errors were in Chapter 6 dealing with the basic properties of HMPs. In section 6.2.3 the description of the Baum–Welch algorithm is not correct. The Baum–Welch algorithm works in conjunction with the forward-backward (BCJR) algorithm [1] and not with the Viterbi algorithm (for the correct version, readers should see either [5, section III.C] or [3, pp. 63–65]). The algorithm, as described in section 6.2.3, is sometimes referred to as Viterbi training [3, pp. 64–65] and is a crude approximation of the Baum–Welch algorithm, which is an instance of expectation maximization [2]. Also, in Chapter 6, equation (6.15) that lists the so-called "backward term" in the forward-backward (BCJR) algorithm is incorrect. The correct equation should be

$$\beta_i(\mathbf{u}, k) = \Pr\left\{(Y_{k+1}, Y_{k+2}, \ldots, Y_l) \mid X_k = i\right\}.$$

Another "less serious but irksome all the same" error is in Example 3.1 in Chapter 3 on nonnegative matrices, where the reachability matrix M in the example does not appear to be correct for the nonnegative (connectivity) matrix A. Furthermore, the text description also appears to be inconsistent with the version of the reachability matrix as provided in the example. Because a number of errors and typos were found upon detailed examination of only selected portions of the book, it is likely that additional errors also exist, several of which may not be easy for readers to immediately identify. Therefore, a useful service to readers would be an errata provided by the author or publisher that is updated periodically.

The coverage of material in the book is quite uneven in multiple aspects and does not appear to clearly align with a specific focus or objective. For instance, it is unclear why the uniqueness of the entropy function is belabored in Chapter 2. This material appears to be drawn directly from Shannon's epic paper [6] and bears no connection at all with the topics covered in the rest of the book. Some of the more elementary chapters and sections feature exercises and abundant examples, whereas there are no exercises and only sparse examples for the advanced topics that are the particular focus of the book, where readers would benefit most from meaningful exercises and examples. As already indicated, the coverage of applications in biology is haphazard and not particularly representative. While the significance and impact of the BLAST algorithm in computational biology cannot be overemphasized, the material in Chapter 9 appears to be an oddball inclusion in the book as it does not relate to HMMs per se—the results used come entirely from the theory of large deviations for i.i.d. sequences. Also, key linkages between topics, and of the material to the central theme of the book, are missing in several places. For instance, the Viterbi estimation and forward-backward recursions for HMMs are also instances of the dynamic programming that was referred to in connection with the heuristic Needleman Wunsch [4] and Smith–Waterman [7] algorithms. Furthermore, by considering scores defined as log-odds ratios of the transition and emission probabilities for HMMs, the pair alignment HMM (which is strangely not included in the book) reduces to an additive score maximization alignment algorithm analogous to the heuristic schemes considered.

The writing style in the book is verbose and desultory. Tangential observations, historical background, and personal opinions are inserted at various random locations within the main thread of presentation instead of being more appropriately organized as notes at the end of each chapter. Currently, many of these observations are also anticipatory and do not necessarily make sense unless the reader has prior familiarity with the topic or until the following material has been read. The writing is not characterized by the frugality of words and precision that is the hallmark of classic mathematical texts. For instance, statements regarding the properties of convex and concave functions, in Chapter 2, are repetitive because of the inclusion of essentially the same results, separately, for both convex and concave functions. Greater economy of space and clarity would be accomplished by simply noting that concave functions are those whose negatives yield convex functions, and then stating results only for convex functions. Another useful feature absent from the book is a table summarizing the notational conventions and acronyms adopted. Such a table would help the reader to keep track of the many different concepts, terms, notation, and acronyms that are used throughout the book, allowing them to better follow results and proofs that are built up sequentially over multiple chapters.

An objective of the book, as stated in the preface, is to provide a treatment of HMPs that includes advanced notions such as ergodicity, representations, analysis of large deviations, and realization of HMPs, while "keeping technicalities to an absolute minimum." In fact, the author states in the preface, "By restricting attention to Markov processes with finite state spaces, I try to capture most of the interesting phenomena such as ergodicity and large deviation theory, while giving elementary proofs that are accessible to anyone who knows undergraduate-level linear algebra." Conceptually, by restricting attention to the setting of a finite state space, the presentation is more broadly accessible, for example, to readers without a background in measure theory. In practice, however, readers of the book will quickly realize that a high level of mathematical maturity is required to follow the material. Readers with only an undergraduate exposure to linear algebra will invariably find it extremely challenging to understand the material. In fact, parts of this review also need to rely on technical terminology that assumes that the reader has prior familiarity with the concepts being discussed.

Overall, the book would have benefited from more careful editing to ensure a clear focus and a terser and more uniform treatment of the material throughout. In its present form, the book appears to be the result of a not-particularly-concerted effort to build a scaffolding for the advanced concepts in the theory of HMPs explored in two of the author's papers [8, 9]. Readers wishing to understand the results of these two papers, who also want the necessary background material to be introduced in context, would benefit most from the current book, whereas those looking for a broader exposition with greater context for theory, practice, and applications in biology will not be particularly well-served.

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Gaurav Sharma
University of Rochester

An Introduction to Mathematical Epidemiology. *By Maia Martcheva.* Springer, New York, 2015. $79.99. xiv+453 pp., hardcover. ISBN 978-1-4899-7611-6.

In 2008, I published an introductory textbook explaining the basics of mathematical epidemiology for a nonexpert audience [1]. It covered simple epidemic models, the basic reproductive ratio (and, in particular, its failure, which was one of the first publications to do so), vector-borne diseases, fitting models to data, and discrete epidemic models. I did so with a lightness of touch with respect to the mathematical details. *An Introduction to Mathematical Epidemiology* covers simple epidemic models, the basic reproductive ratio (and, in particular, its failure), vector-borne diseases, fitting models to data, and discrete epidemic models. To be fair, it also covers global stability, multistrain disease dynamics, optimal control, age- and class-structure, and immuno-epidemic modeling.

This isn't necessarily a problem, as there's utility in presenting these topics with the mathematical details filled in, as in the first half of the book. The new material is also worthy of consideration. It's unacceptable that the source material isn't referenced at all, however. There are even statements like "some researchers believe that it should not be called a reproduction number" (p. 110) without any attributions.

In general, the book presents a variety of ordinary differential equation (ODE) models. These are fine, although it does become a bit samey after a while. However, the models chosen are somewhat simplistic. In particular, a variety of deadly diseases are modeled—e.g., malaria (p. 70) or TB (p. 165)—with the assumption that there is no death rate due to disease. This appears to be done so that the models are more tractable, but they ignore the biological reality of the situation. Indeed, the concept of a disease-specific death rate is not introduced in any of the models until over halfway through. This is highly unrealistic.

The definition of chaos is missing topological transitivity (p. 81), which is entirely misleading: wholly unstable systems are also aperiodic and have sensitive dependence on initial conditions, but they aren't chaotic. There are also numerous issues with the proofreading, such as comma splices (e.g., pp. 116, 224, 389, 422) and glaring typos like "is always grater than" (p. 153). Sloppiness like this and the lack of referencing makes this book far less useful than it otherwise might be.

Where the book excels, however, is when it gives us the MATLAB code (p. 127) and resources for finding data (p. 125) and model fitting (p. 129). This is superb. Students can input their own code and can start digging for real-world data to use to fit their models. The twelve pages that deal with this are the undoubted highlight of the book. More like this would have been excellent.

In summary, this is a book that means well but has significant flaws. The sameness of each chapter containing what are essentially small variations on the basic ODE models makes it a slog to read at over 400 pages. There are some variations and some more MATLAB code in the last few chapters, but it's too little, too late. It's unclear who the audience for this book would be; mathematicians surely want more challenges than just simple ODEs, whereas biologists would be unlikely to wade their way through all the mathematical details. However, the material on data fitting is excellent.

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Robert Smith?
The University of Ottawa

Certified Reduced Basis Methods for Parametrized Partial Differential Equations. *By Jan S. Hesthaven, Gianluigi Rozza, and Benjamin Stamm.* Springer, Cham, Switzerland, 2016. $69.99. xiv+131 pp., softcover. ISBN 978-3-319-22469-5.

One of the most exciting and fascinating challenges of numerical mathematics for the next several years will be to bring the power of modeling and simulations to real-world engineering problems (broadly understood) within short timelines. Ideally, one would like to have almost real-time quantitative responses for problems that require complex methods of solution. Having more powerful hardware is clearly not sufficient to address the demands coming not only from the engineering world but also from medicine and clinics, to mention just one application, since high-performance computing facilities are not necessarily easily accessible. The answer to this demand is likely to come from a combination of computer science and mathematics ingredients—infrastructure and methods, hardware, and software.

In this scenario, an extremely promising and effective tool arising from the mathematical side for solving partial differential equations depending on one or more parameters—as invariably happens in real applications—is provided by the *reduced basis method*, originally developed in the groups of T. Patera and Y. Maday. Traditional Galerkin methods (like finite elements) for representing the solution of a partial differential equation pursue a sort of "general purpose" approach, where the approximate solution is represented by means of a generic (piecewise polynomial, but also trigonometric or exponential) set of functions not designed for a specific problem, but rather useful for virtually countless differential problems. This versatility implies that all the problem-specific information is carried by the coefficients of the finite-dimensional expansion with respect to the selected set of basis functions. This generally requires many degrees of freedom. Consequently, the numerical problems feature large dimensions and eventually high computational costs. The reduced basis method still relies on the Galerkin framework, following the idea that the selected basis function set is customized or "educated" about the problem to be solved. The problem-specific information is captured by both the basis functions and the coefficients. Giving up versatility may reduce the number of degrees of freedom required for the reliable approximation of the problem at hand, with a computational advantage. This may be useful in the extremely common situation of parametrized partial differential equations. As a matter of fact, in modern engineering the challenge is quite often the identification of those parameters or the detection of their optimal values for the minimization (maximization) of a cost (benefit) functional. The development of a method that can rapidly evaluate the solution for specific values of the parameters is therefore critical. In the reduced basis method, the idea is to inform the basis functions about the differential nature of the problem at hand and to leave to the coefficients the task of specifying the solution for a desired parameter value, following an offline/online paradigm. In the offline step the basis function is constructed by solving the problem (analytically or, more commonly, with a general purpose traditional method) for a given set of values of the parameter(s). This idea is brilliant and has found many applications, yet there are many issues to address and methodological details to understand. Until now, a book providing a compact but comprehensive introduction to the topic was lacking. The book under review fills this gap.

The introduction (Chapter 1) draws a broad outline of the topic, including historical notes. A remarkable feature is the annotated list of specialized software libraries implementing the reduced basis method.

In Chapter 2 is given a description of the basic ingredients of the method. In particular, the general notion of parametrized PDEs, their weak formulation, and the basic mathematical assumptions used in the book are clearly stated. General purpose discretization techniques are rapidly introduced. Two two-dimensional examples are presented as reference problems used in the subsequent chapters. The first one concerns a classical (steady) heat equation where the thermal conductivity is piecewise constant in two regions. The parameters are the two values of the thermal conductivity and the heat flux on the basis of the domain of interest. The second example is based on a linear elasticity problem, where the parameters are the eight Young moduli in different parts of the region of interest and the three horizontal loads at the right border of the domain.

Chapter 3 addresses the construction of the offline basis, following both the proper orthogonal decomposition (POD) strategy and the greedy approach typical of reduced basis methods based on a posteriori error estimators. The concept of Kolmogorov N-width is clearly pivotal here, as it defines how a problem can be effectively represented by low-dimensional models. The importance of the *affine decomposition* assumption for the dependence of the solution on the parameters is discussed and eventually demonstrated on the two benchmarks introduced above. The construction of the reduced basis requires the computation of the solution of the differential equation (the truth) for different values of the parameters. This forms the basis of the reduced approximation. In the greedy strategy, the selection of parameters finds the values that maximize a suitable a posteriori error estimator. The definition of these error bounds is addressed in detail in Chapter 4. This is a technical chapter, yet critical for an understanding of the different ways of evaluating the error in the selection of the values of the parameters. Different bounds (viz. different norms) are considered and their actual computation is also addressed. Again, the application to the two examples mentioned above is carried out.

One of the most serious limitations of the original method is the assumption of affine dependence of the solution on the parameters. Unfortunately, this type of dependence does not occur in most of the problems of real interest. Specific techniques are needed to extend the original idea to nonaffine problems without losing the computational efficiency. This extension relies on the so-called *empirical interpolation method* (EIM) described in Chapter 5. In particular, the heat conduction problem is modified, first with a parametrization of the geometry that makes the problem not affine, and then with the introduction of nonlinear terms. The so-called discrete EIM (DEIM) is described too. The DEIM is based on a POD procedure rather than on a greedy one, as is discussed in the final section of the chapter.

Chapter 6 is a perspective conclusion of the work where the reduced basis method is extended to:

(i) time-dependent problems (with a combination of POD-greedy approaches for the construction of the basis);

(ii) noncompliant problems, i.e., problems where the output functional is generic (as opposed to the case when the output is just the forcing term of the differential problem applied to the solution);

(iii) problems where the parameters refer to the geometry;

(iv) noncoercive problems, including elasticity and incompressible fluid dynamics.

Illustrative examples are presented by properly adjusting the original heat conduction problem.

The presentation is completed by two appendices containing background material.

One of the distinctive features of the book is the presence of text boxes (i.e., special text areas with a gray background) addressing more practical issues related to the implementation and the solution of linear algebra problems. These boxes should be extremely useful for readers more interested in the "educated implementation" of these methods than in their theoretical framework.

Overall, this book fills an important gap in the literature. It is an excellent starting point for young scholars approaching this new world, since it gives a well-balanced combination of theoretical aspects, practical details, and examples. Technical details in these topics may be sometimes overwhelming for new students or engineers who just want to be introduced to the methods. The book should be enjoyable for those readers thanks to its several examples and a generally straightforward description of the methods. It also gives a nice picture of the research in this field, particularly in the last two chapters which address advanced (yet crucial) topics.

Alessandro Veneziani
Emory University

Discrete Fourier and Wavelet Transforms: An Introduction through Linear Algebra with Applications to Signal Processing. *By Roe W. Goodman.* World Scientific, Singapore, 2016. $48.00. xii+288 pp., softcover. ISBN 978-981-4725-77-4.

This textbook arose from a course taught by the author to undergraduates at Rutgers University over a number of years. By restricting his attention to the discrete (as opposed to continuous) Fourier and wavelet transforms, the author was able to offer a course with relatively few prerequisites and thereby reach a broader audience. The mathematics involved is almost all linear algebra.

There are five chapters, each of which ends with a substantial section called "Computer Explorations" in MATLAB followed by a good set of pencil-and-paper exercises. The computer explorations are keyed to specific sections of the book, so the student does not have to read all the way to the end of the chapter before getting started on them.

Chapter 1 reviews the linear algebra prerequisites and also has a short section on Fourier series to help motivate the introduction of the discrete Fourier transform. Chapter 2 begins with a discussion of sampling and aliasing, then introduces the discrete Fourier transform and its matrix representation. This is shown to diagonalize circulant matrices and therefore turn circular convolution (filtering) in the time domain into multiplication in the frequency domain. Finally, the fast Fourier transform is presented.

Chapter 3 begins the discussion of wavelet transforms. The author's strategy is to introduce several examples as quickly as possible with minimal explanation by the lifting method. Of course, the Haar wavelet transform comes first, then CDF(2,2) and Daub4. The explanation of the Haar transform and how it stores information at different resolutions is reasonably clear, but CDF(2,2) and Daub4 are simply presented with no explanation of why they might be useful. (All theory is postponed to Chapter 4.) The student must take a lot on faith at this juncture. The payoff is that the student can get started right away with computer experimentation on a variety of simple wavelet transforms and can discover by direct experience that some do a better job of signal compression than others. Toward the end of Chapter 3, two-dimensional wavelets for image compression are introduced.

Chapter 4 presents the theory. Wavelets come from filter banks. The signal goes through a low-pass filter and a complementary high-pass filter and then gets decimated, etc. The z-transform moves the analysis to the frequency domain, and gradually we find out what constitutes a good wavelet.

The final chapter, which is not a part of the author's course, gives the reader a glimpse of the continuous theory: wavelet transforms for analog signals.

There is no doubt that the diligent reader can learn a lot from this book. However, I do have a few complaints. The author tends to use jargon without explanation, apparently expecting the student to gain understanding by osmosis. For example, the terms *prediction* and *update* are used in Chapter 3 without any indication of why these are appropriate names for these steps. Definitions, examples, lemmas, theorems, propositions, and remarks are all numbered by separate counters, and they are numbered by chapter, not by section. This makes navigation difficult. The index is rather sparse.

These minor gripes aside, this is a compact and inexpensive book that gives a relatively quick introduction to discrete wavelet theory.

DAVID S. WATKINS
Washington State University